FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Physics Grades 10 - 12

Version 0
November 9, 2008

STOP!!!!

Did you notice the **FREEDOMS** we've granted you?

Our copyright license is **different!** It grants freedoms rather than just imposing restrictions like all those other textbooks you probably own or use.

- We know people copy textbooks illegally but we would LOVE it if you copied our's - go ahead copy to your hearts content, **legally!**
- Publishers' revenue is generated by controlling the market, we don't want any money, go ahead, distribute our books far and wide - we DARE you!
- Ever wanted to change your textbook? Of course you have! Go ahead, change ours, make your own version, get your friends together, rip it apart and put it back together the way you like it. That's what we really want!
- Copy, modify, adapt, enhance, share, critique, adore, and contextualise. Do it all, do it with your colleagues, your friends, or alone but get involved! Together we can overcome the challenges our complex and diverse country presents.
- So what is the catch? The only thing you can't do is take this book, make a few changes and then tell others that they can't do the same with your changes. It's share and share-alike and we know you'll agree that is only fair.
- These books were written by volunteers who want to help support education, who want the facts to be freely available for teachers to copy, adapt and re-use. Thousands of hours went into making them and they are a gift to everyone in the education community.

FHSST Core Team

Mark Horner ; Samuel Halliday ; Sarah Blyth ; Rory Adams ; Spencer Wheaton

FHSST Editors

Jaynie Padayachee ; Joanne Boulle ; Diana Mulcahy ; Annette Nell ; René Toerien ; Donovan Whitfield

FHSST Contributors

Rory Adams ; Prashant Arora ; Richard Baxter ; Dr. Sarah Blyth ; Sebastian Bodenstein ; Graeme Broster ; Richard Case ; Brett Cocks ; Tim Crombie ; Dr. Anne Dabrowski ; Laura Daniels ; Sean Dobbs ; Fernando Durrell ; Dr. Dan Dwyer ; Frans van Eeden ; Giovanni Franzoni ; Ingrid von Glehn ; Tamara von Glehn ; Lindsay Glesener ; Dr. Vanessa Godfrey ; Dr. Johan Gonzalez ; Hemant Gopal ; Umeshree Govender ; Heather Gray ; Lynn Greeff ; Dr. Tom Gutierrez ; Brooke Haag ; Kate Hadley ; Dr. Sam Halliday ; Asheena Hanuman ; Neil Hart ; Nicholas Hatcher ; Dr. Mark Horner ; Robert Hovden ; Mfandaidza Hove ; Jennifer Hsieh ; Clare Johnson ; Luke Jordan ; Tana Joseph ; Dr. Jennifer Klay ; Lara Kruger ; Sihle Kubheka ; Andrew Kubik ; Dr. Marco van Leeuwen ; Dr. Anton Machacek ; Dr. Komal Maheshwari ; Kosma von Maltitz ; Nicole Masureik ; John Mathew ; JoEllen McBride ; Nikolai Meures ; Riana Meyer ; Jenny Miller ; Abdul Mirza ; Asogan Moodaly ; Jothi Moodley ; Nolene Naidu ; Tyrone Negus ; Thomas O'Donnell ; Dr. Markus Oldenburg ; Dr. Jaynie Padayachee ; Nicolette Pekeur ; Sirika Pillay ; Jacques Plaut ; Andrea Prinsloo ; Joseph Raimondo ; Sanya Rajani ; Prof. Sergey Rakityansky ; Alastair Ramlakan ; Razvan Remsing ; Max Richter ; Sean Riddle ; Evan Robinson ; Dr. Andrew Rose ; Bianca Ruddy ; Katie Russell ; Duncan Scott ; Helen Seals ; Ian Sherratt ; Roger Sieloff ; Bradley Smith ; Greg Solomon ; Mike Stringer ; Shen Tian ; Robert Torregrosa ; Jimmy Tseng ; Helen Waugh ; Dr. Dawn Webber ; Michelle Wen ; Dr. Alexander Wetzler ; Dr. Spencer Wheaton ; Vivian White ; Dr. Gerald Wigger ; Harry Wiggins ; Wendy Williams ; Julie Wilson ; Andrew Wood ; Emma Wormauld ; Sahal Yacoob ; Jean Youssef

Contributors and editors have made a sincere effort to produce an accurate and useful resource. Should you have suggestions, find mistakes or be prepared to donate material for inclusion, please don't hesitate to contact us. We intend to work with all who are willing to help make this a continuously evolving resource!

www.fhsst.org

Contents

Chapter 26

2D and 3D Wavefronts - Grade 12

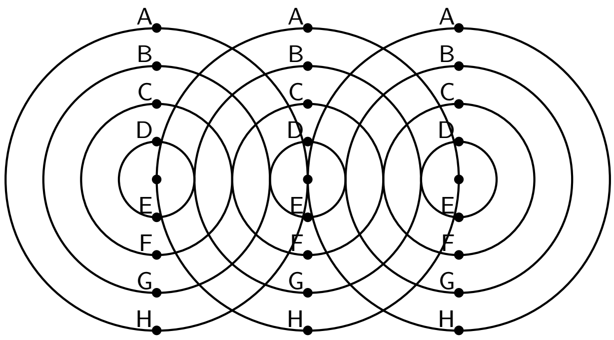
26.1 Introduction

You have learnt about the basic principles of reflection and refraction. In this chapter, you will learn about phenomena that arise with waves in two and three dimensions: interference and diffraction.

26.2 Wavefronts

Activity :: Investigation : Wavefronts

The diagram shows three identical waves being emitted by three point sources. All points marked with the same letter are in phase. Join all points with the same letter.



What type of lines (straight, curved, etc) do you get? How does this compare to the line that joins the sources?

Consider three point sources of waves. If each source emits waves isotropically (i.e. the same in all directions) we will get the situation shown in as shown in Figure 26.1.

We define a **wavefront** as the imaginary line that joins waves that are in phase. These are indicated by the grey, vertical lines in Figure 26.1. The points that are in phase can be peaks, troughs or anything in between, it doesn't matter which points you choose as long as they are in phase.

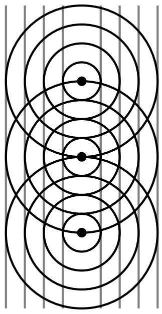


Figure 26.1: Wavefronts are imaginary lines joining waves that are in phase. In the example, the wavefronts (shown by the grey, vertical lines) join all waves at the crest of their cycle.

26.3 The Huygens Principle

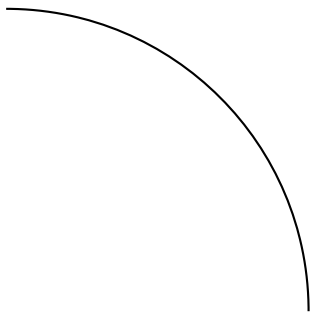
Christiaan Huygens described how to determine the path of waves through a medium.

Definition: The Huygens Principle
Each point on a wavefront acts like a point source of circular waves. The waves emitted from these point sources interfere to form another wavefront.

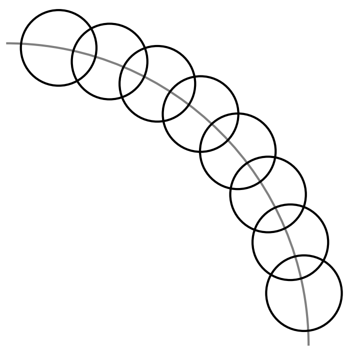
A simple example of the Huygens Principle is to consider the single wavefront in Figure 26.2.

Worked Example 169: Application of the Huygens Principle

Question: Given the wavefront,



use the Huygens Principle to determine the wavefront at a later time.
Answer
Step 1 : Draw circles at various points along the given wavefront



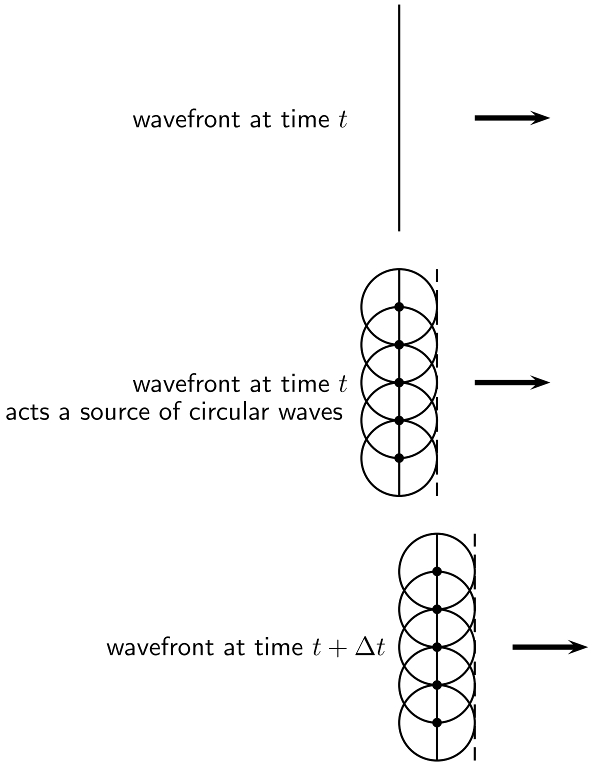
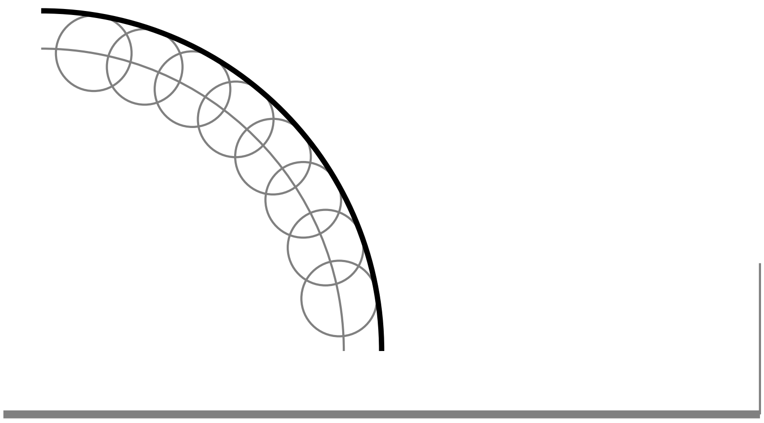


Figure 26.2: A single wavefront at time t acts as a series of point sources of circular waves that interfere to give a new wavefront at a time $t + \Delta t$. The process continues and applies to any shape of waveform.

Step 2 : Join the crests of each circle to get the wavefront at a later time



Interesting Fact

Christiaan Huygens (14 April 1629 - 8 July 1695), was a Dutch mathematician, astronomer and physicist; born in The Hague as the son of Constantijn Huygens. He studied law at the University of Leiden and the College of Orange in Breda before turning to science. Historians commonly associate Huygens with the scientific revolution.

Huygens generally receives minor credit for his role in the development of modern calculus. He also achieved note for his arguments that light consisted of waves; see: wave-particle duality. In 1655, he discovered Saturn's moon Titan. He also examined Saturn's planetary rings, and in 1656 he discovered that those rings consisted of rocks. In the same year he observed and sketched the Orion Nebula. He also discovered several interstellar nebulae and some double stars.

26.4 Interference

Interference occurs when two identical waves pass through the same region of space at the same time resulting in a superposition of waves. There are two types of interference which is of interest: **constructive** interference and **destructive** interference.

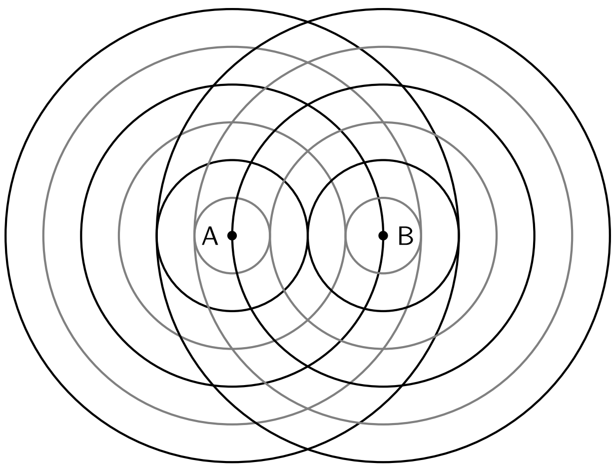
Constructive interference occurs when both waves have a displacement in the same direction, while destructive interference occurs when one wave has a displacement in the opposite direction to the other, thereby resulting in a cancellation. There is no displacement of the medium in destructive interference while for constructive interference the displacement of the medium is greater than the individual displacements.

Constructive interference occurs when both waves have a displacement in the same direction, this means they both have a peak or they both have a trough at the same place at the same time. If they both have a peak then the peaks add together to form a bigger peak. If they both have a trough then the trough gets deeper.

Destructive interference occurs when one wave has a displacement in the opposite direction to the other, this means that the one wave has a peak and the other wave has a trough. If the waves have identical magnitudes then the peak "fills" up the trough and the medium will look like there are no waves at that point. There will be no displacement of the medium. A place where destructive interference takes places is called a node.

Waves can interfere at places where there is never a trough and trough or peak and peak or trough and peak at the same time. At these places the waves will add together and the resultant displacement will be the sum of the two waves but they won't be points of maximum interference.

Consider the two identical waves shown in the picture below. The wavefronts of the peaks are shown as black lines while the wavefronts of the troughs are shown as grey lines. You can see that the black lines cross other black lines in many places. This means two peaks are in the same place at the same time so we will have constructive interference where the two peaks add together to form a bigger peak.



Two points sources (A and B) radiate identical waves. The wavefronts of the peaks (black lines) and troughs (grey lines) are shown. Constructive interference occurs where two black lines intersect or where two gray lines intersect. Destructive interference occurs where a black line intersects with a grey line.

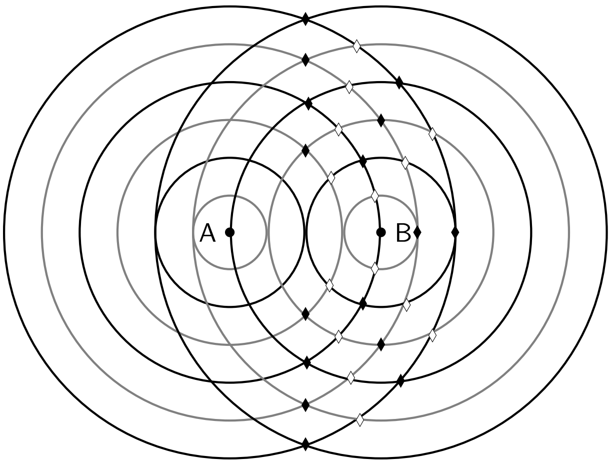
You can see that the black lines cross other black lines in many places. This means two peaks are in the same place at the same time so we will have constructive interference where the two peaks add together to form a bigger peak.

When the grey lines cross other grey lines there are two troughs are in the same place at the same time so we will have constructive interference where the two troughs add together to form a bigger trough.

In the case where a grey line crosses a black line we are seeing a trough and peak in the same place. These will cancel each other out and the medium will have no displacement at that point.

- black line + black line = peak + peak = constructive interference
- grey line + grey line = trough + trough = constructive interference
- black line + grey line = grey line + black line = peak + trough = trough + peak = destructive interference

On half the picture below, we have marked the constructive interference with a solid black diamond and the destructive interference with a hollow diamond.



To see if you understand it, cover up the half we have marked with diamonds and try to work out which points are constructive and destructive on the other half of the picture. The two halves are mirror images of each other so you can check yourself.

26.5 Diffraction

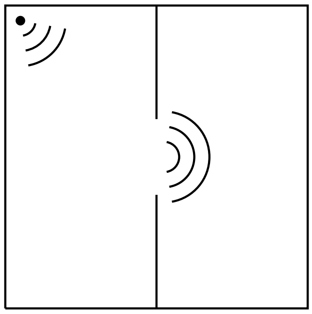
One of the most interesting, and also very useful, properties of waves is **diffraction**.

Definition: Diffraction
Diffraction is the ability of a wave to spread out in wavefronts as the wave passes through a small aperture or around a sharp edge.

Extension: Diffraction

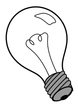
Diffraction refers to various phenomena associated with wave propagation, such as the bending, spreading and interference of waves emerging from an aperture. It occurs with any type of wave, including sound waves, water waves, electromagnetic waves such as light and radio waves. While diffraction always occurs, its effects are generally only noticeable for waves where the wavelength is on the order of the feature size of the diffracting objects or apertures.

For example, if two rooms are connected by an open doorway and a sound is produced in a remote corner of one of them, a person in the other room will hear the sound as if it originated at the doorway.



As far as the second room is concerned, the vibrating air in the doorway is the source of the sound. The same is true of light passing the edge of an obstacle, but this is not as easily observed because of the short wavelength of visible light.

This means that when waves move through small holes they appear to bend around the sides because there are not enough points on the wavefront to form another straight wavefront. This is bending round the sides we call *diffraction*.



Extension: Diffraction

Diffraction effects are more clear for water waves with longer wavelengths. Diffraction can be demonstrated by placing small barriers and obstacles in a ripple tank and observing the path of the water waves as they encounter the obstacles. The waves are seen to pass around the barrier into the regions behind it; subsequently the water behind the barrier is disturbed. The amount of diffraction (the sharpness of the bending) increases with increasing wavelength and decreases with decreasing wavelength. In fact, when the wavelength of the waves are smaller than the obstacle, no noticeable diffraction occurs.

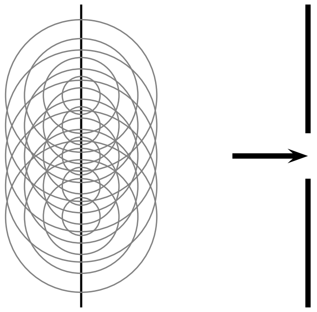
Activity :: Experiment : Diffraction

Water waves in a ripple tank can be used to demonstrate diffraction and interference.

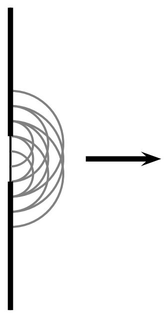
26.5.1 Diffraction through a Slit

When a wave strikes a barrier with a hole only part of the wave can move through the hole. If the hole is similar in size to the wavelength of the wave diffractions occurs. The waves that comes through the hole no longer looks like a straight wave front. It bends around the edges of the hole. If the hole is small enough it acts like a point source of circular waves.

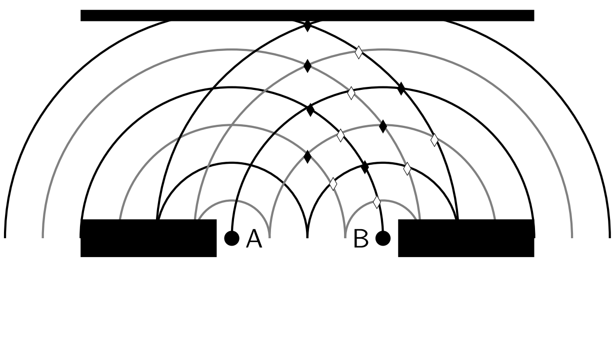
Now if allow the wavefront to impinge on a barrier with a hole in it, then only the points on the wavefront that move into the hole can continue emitting forward moving waves - but because a lot of the wavefront have been removed the points on the edges of the hole emit waves that bend round the edges.



If you employ Huygens' principle you can see the effect is that the wavefronts are no longer straight lines.

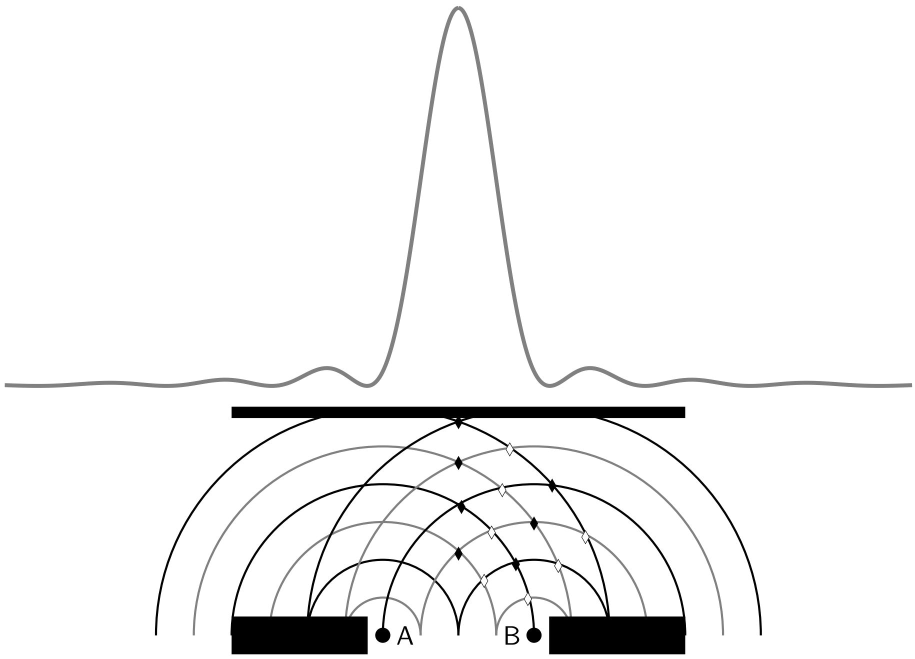


Each point of the slit acts like a point source. If we think about the two point sources on the edges of the slit and call them A and B then we can go back to the diagram we had earlier but with some parts block by the wall.



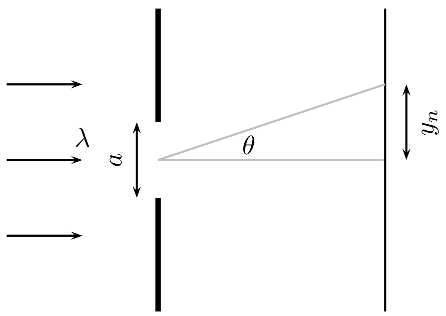
If this diagram were showing sound waves then the sound would be louder (constructive interference) in some places and quieter (destructive interference) in others. You can start to see that there will be a pattern (interference pattern) to the louder and quieter places. If we were studying light waves then the light would be brighter in some places than others depending on the interferences.

The intensity (how bright or loud) of the interference pattern for a single narrow slit looks like this:



The picture above shows how the waves add together to form the interference pattern. The peaks correspond to places where the waves are adding most intensely and the zeroes are places where destructive interference is taking place. When looking at interference patterns from light the spectrum looks like:

There is a formula we can use to determine where the peaks and minimums are in the interference spectrum. There will be more than one minimum. There are the same number of minima on either side of the central peak and the distances from the first one on each side are the same to the peak. The distances to the peak from the second minimum on each side is also the same, in fact the two sides are mirror images of each other. We label the first minimum that corresponds to a positive angle from the centre as $m = 1$ and the first on the other side (a negative angle from the centre as $m = -1$, the second set of minima are labelled $m = 2$ and $m = -2$ etc.



The equation for the angle at which the minima occur is

Definition: Interference Minima

The angle at which the minima in the interference spectrum occur is:

$$\sin\theta = \frac{m\lambda}{a}$$

where

θ is the angle to the minimum

λ is the wavelength of the impinging wavefronts

m is the order of the mimimum, $m = \pm 1, \pm 2, \pm 3, ...$

Worked Example 170: Diffraction Minimum I

Question: A slit has a width of 2511 nm has red light of wavelength 650 nm impinge on it. The diffracted light interferers on a surface, at what angle will the first minimum be?

Answer

Step 1 : Check what you are given

We know that we are dealing with interference patterns from the diffraction of light passing through a slit. The slit has a width of 2511 nm which is 2511×10^{-9} m and we know that the wavelength of the light is 650 nm which is 650×10^{-9} m. We are looking to determine the angle to first minimum so we know that $m = 1$.

Step 2 : Applicable principles

We know that there is a relationship between the slit width, wavelength and interference minimum angles:

$$\sin\theta = \frac{m\lambda}{a}$$

We can use this relationship to find the angle to the minimum by substituting what we know and solving for the angle.

Step 3 : Substitution

$$\begin{aligned}
\sin\theta &= \frac{650 \times 10^{-9}}{2511 \times 10^{-9}} \\
\sin\theta &= \frac{650}{2511} \\
\sin\theta &= 0.258861012 \\
\theta &= \sin^{-1} 0.258861012 \\
\theta &= 15^{o}
\end{aligned}$$

The first minimum is at 15 degrees from the centre peak.

Worked Example 171: Diffraction Minimum II

Question: A slit has a width of 2511 nm has green light of wavelength 532 nm impinge on it. The diffracted light interferers on a surface, at what angle will the first minimum be?

Answer

Step 1 : Check what you are given

We know that we are dealing with interference patterns from the diffraction of light passing through a slit. The slit has a width of 2511 nm which is 2511×10^{-9} m and we know that the wavelength of the light is 532 nm which is 532×10^{-9} m. We are looking to determine the angle to first minimum so we know that $m = 1$.

Step 2 : Applicable principles

We know that there is a relationship between the slit width, wavelength and interference minimum angles:

$$\sin\theta = \frac{m\lambda}{a}$$

We can use this relationship to find the angle to the minimum by substituting what we know and solving for the angle.

Step 3 : Substitution

$$\begin{aligned}\sin\theta &= \frac{532\times10^{-9}}{2511\times10^{-9}}\\ \sin\theta &= \frac{532}{2511}\\ \sin\theta &= 0.211867782\\ \theta &= \sin^{-1}0.211867782\\ \theta &= 12.2^{o}\end{aligned}$$

The first minimum is at 12.2 degrees from the centre peak.

From the formula you can see that a smaller wavelength for the same slit results in a smaller angle to the interference minimum. This is something you just saw in the two worked examples. Do a sanity check, go back and see if the answer makes sense. Ask yourself which light had the longer wavelength, which light had the larger angle and what do you expect for longer wavelengths from the formula.

Worked Example 172: Diffraction Minimum III

Question: A slit has a width which is unknown and has green light of wavelength 532 nm impinge on it. The diffracted light interferers on a surface, and the first minimum is measure at an angle of 20.77 degrees?

Answer

Step 1 : Check what you are given

We know that we are dealing with interference patterns from the diffraction of light passing through a slit. We know that the wavelength of the light is 532 nm which is 532×10^{-9} m. We know the angle to first minimum so we know that $m = 1$ and $\theta = 20.77^{o}$.

Step 2 : Applicable principles

We know that there is a relationship between the slit width, wavelength and interference minimum angles:

$$\sin\theta = \frac{m\lambda}{a}$$

We can use this relationship to find the width by substituting what we know and solving for the width.

Step 3 : Substitution

$$\begin{aligned}\sin\theta &= \frac{532\times10^{-9}}{a}\\ \sin 20.77 &= \frac{532\times10^{-9}}{a}\\ a &= \frac{532\times10^{-9}}{0.354666667}\\ a &= 1500\times10^{-9}\\ a &= 1500\ \text{nm}\end{aligned}$$

The slit width is 1500 nm.

26.6 Shock Waves and Sonic Booms

Now we know that the waves move away from the source at the speed of sound. What happens if the source moves at the same time as emitting sounds? Once a sound wave has been emitted it is no longer connected to the source so if the source moves it doesn't change the way the sound wave is propagating through the medium. This means a source can actually catch up with a sound waves it has emitted.

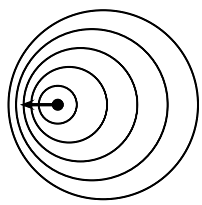
The speed of sound is very fast in air, about $340\ \mathrm{m \cdot s^{-1}}$, so if we want to talk about a source catching up to sound waves then the source has to be able to move very fast. A good source of sound waves to discuss is a jet aircraft. Fighter jets can move very fast and they are very noisy so they are a good source of sound for our discussion. Here are the speeds for a selection of aircraft that can fly faster than the speed of sound.

Aircraft	speed at altitude ($\mathrm{km \cdot h^{-1}}$)	speed at altitude ($\mathrm{m \cdot s^{-1}}$)
Concorde	2 330	647
Gripen	2 410	669
Mirage F1	2 573	990
Mig 27	1 885	524
F 15	2 660	739
F 16	2 414	671

26.6.1 Subsonic Flight

Definition: Subsonic
Subsonic refers to speeds slower than the speed of sound.

When a source emits sound waves and is moving but slower than the speed of sound you get the situation in this picture. Notice that the source moving means that the wavefronts and therefore peaks in the wave are actually closer together in the one direction and further apart in the other.



subsonic flight

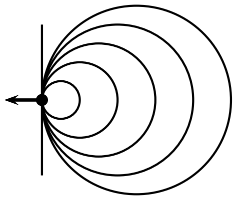
If you measure the waves on the side where the peaks are closer together you'll measure a different wavelength than on the other side of the source. This means that the noise from the source will sound different on the different sides. This is called the *Doppler Effect*.

Definition: Doppler Effect
when the wavelength and frequency measured by an observer are different to those emitted by the source due to movement of the source or observer.

26.6.2 Supersonic Flight

Definition: Supersonic
Supersonic refers to speeds faster than the speed of sound.

If a plane flies at exactly the speed of sound then the waves that it emits in the direction it is flying won't be able to get away from the plane. It also means that the next sound wave emitted will be exactly on top of the previous one, look at this picture to see what the wavefronts would look like:



shock wave at Mach 1

Sometimes we use the speed of sound as a reference to describe the speed of the object (aircraft in our discussion).

Definition: Mach Number
The Mach Number is the ratio of the speed of an object to the speed of sound in the surrounding medium.

Mach number is tells you how many times faster than sound the aircraft is moving.

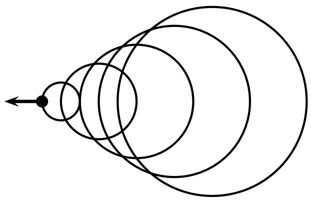
- Mach Number < 1 : aircraft moving slower than the speed of sound
- Mach Number $= 1$: aircraft moving at the speed of sound
- Mach Number > 1 : aircraft moving faster than the speed of sound

To work out the Mach Number divide the speed of the aircraft by the speed of sound.

$$\text{Mach Number} = \frac{\text{v}_{\text{aircraft}}}{\text{v}_{\text{sound}}}$$

Remember: the units must be the same before you divide.

If the aircraft is moving faster than the speed of sound then the wavefronts look like this:



supersonic shock wave

If the source moves faster than the speed of sound a cone of wave fronts is created. This is called a Mach cone. From constructive interference we know that two peaks that add together form a larger peak. In a Mach cone many, many peaks add together to form a very large peak, this is a sound wave so the large peak is a very very loud sound wave. This sounds like a huge "boom" and we call the noise a *sonic boom*.

Worked Example 173: Mach Speed I

Question: An aircraft flies at 1300 $\text{km} \cdot \text{h}^{-1}$ and the speed of sound in air is $340\ \text{m} \cdot \text{s}^{-1}$. What is the Mach Number of the aircraft?
Answer
Step 1 : Check what you are given

We know we are dealing with Mach Number. We are given the speed of sound in air, $340\ \mathrm{m \cdot s^{-1}}$, and the speed of the aircraft, $1300\ \mathrm{km \cdot h^{-1}}$. The speed of the aircraft is in different units to the speed of sound so we need to convert the units:

$$\begin{aligned} 1300\mathrm{km \cdot h^{-1}} &= 1300\mathrm{km \cdot h^{-1}} \\ 1300\mathrm{km \cdot h^{-1}} &= 1300 \times \frac{1000\mathrm{m}}{3600\mathrm{s}} \\ 1300\mathrm{km \cdot h^{-1}} &= 361.1\ \mathrm{m \cdot s^{-1}} \end{aligned}$$

Step 2 : Applicable principles

We know that there is a relationship between the Mach Number, the speed of sound and the speed of the aircraft:

$$\text{Mach Number} = \frac{\mathrm{v_{aircraft}}}{\mathrm{v_{sound}}}$$

We can use this relationship to find the Mach Number.

Step 3 : Substitution

$$\begin{aligned} \text{Mach Number} &= \frac{v_{aircraft}}{v_{sound}} \\ \text{Mach Number} &= \frac{361.1}{340} \\ \text{Mach Number} &= 1.06 \end{aligned}$$

The Mach Number is 1.06.

Definition: Sonic Boom

A sonic boom is the sound heard by an observer as a shockwave passes.

Exercise: Mach Number

In this exercise we will determine the Mach Number for the different aircraft in the table mentioned above. To help you get started we have calculated the Mach Number for the Concord with a speed of sound $v_{sound} = 340\ \mathrm{ms^{-1}}$.

For the Condorde we know the speed and we know that:

$$\text{Mach Number} = \frac{\mathrm{v_{aircraft}}}{\mathrm{v_{sound}}}$$

For the Concorde this means that

$$\begin{aligned} \text{Mach Number} &= \frac{647}{340} \\ &= 1.9 \end{aligned}$$

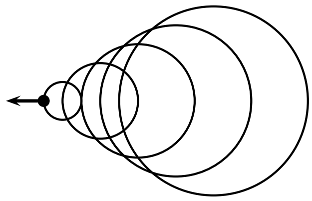
Aircraft	speed at altitude $(\mathrm{km \cdot h^{-1}})$	speed at altitude $(\mathrm{m \cdot s^{-1}})$	Mach Number
Concorde	2 330	647	1.9
Gripen	2 410	669	
Mirage F1	2 573	990	
Mig 27	1 885	524	
F 15	2 660	739	
F 16	2 414	671	

Now calculate the Mach Numbers for the other aircraft in the table.

26.6.3 Mach Cone

You can see that the shape of the Mach Cone depends on the speed of the aircraft. When the Mach Number is 1 there is no cone but as the aircraft goes faster and faster the angle of the cone gets smaller and smaller.

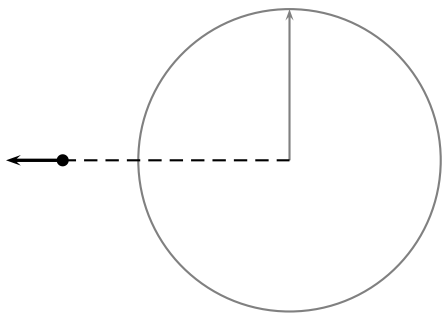
If we go back to the supersonic picture we can work out what the angle of the cone must be.



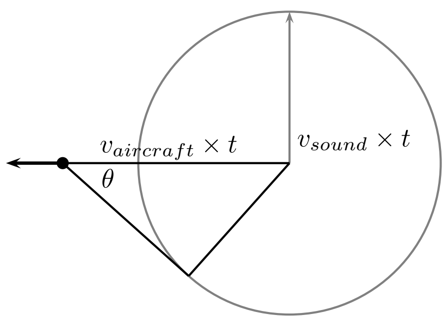
supersonic shock wave

We build a triangle between how far the plane has moved and how far a wavefront at right angles to the direction the plane is flying has moved:

An aircraft emits a sound wavefront. The wavefront moves at the speed of sound 340 $\mathrm{m \cdot s^{-1}}$ and the aircraft moves at Mach 1.5, which is $1.5 \times 340 = 510\ \mathrm{m \cdot s^{-1}}$. The aircraft travels faster than the wavefront. If we let the wavefront travel for a time t then the following diagram will apply:



We know how fast the wavefront and the aircraft are moving so we know the distances that they have traveled:



The angle between the cone that forms at the direction of the plane can be found from the right-angle triangle we have drawn into the figure. We know that $\sin\theta = \frac{opposite}{hypotenuse}$ which in this figure means:

$$\begin{aligned}\sin\theta &= \frac{opposite}{hypotenuse}\\ \sin\theta &= \frac{v_{sound}\times t}{v_{aircraft}\times t}\\ \sin\theta &= \frac{v_{sound}}{v_{aircraft}}\end{aligned}$$

In this case we have used sound and aircraft but a more general way of saying this is:

- aircraft = source
- sound = wavefront

We often just write the equation as:

$$\begin{aligned}\sin\theta &= \frac{v_{sound}}{v_{aircraft}}\\ v_{aircraft}\sin\theta &= v_{sound}\\ v_{source}\sin\theta &= v_{wavefront}\\ v_s\sin\theta &= v_w\end{aligned}$$

?

Exercise: Mach Cone

In this exercise we will determine the Mach Cone Angle for the different aircraft in the table mentioned above. To help you get started we have calculated the Mach Cone Angle for the Concorde with a speed of sound $v_{sound} = 340\ \mathrm{m\cdot s^{-1}}$.

For the Condorde we know the speed and we know that:

$$\sin\theta = \frac{v_{sound}}{v_{aircraft}}$$

For the Concorde this means that

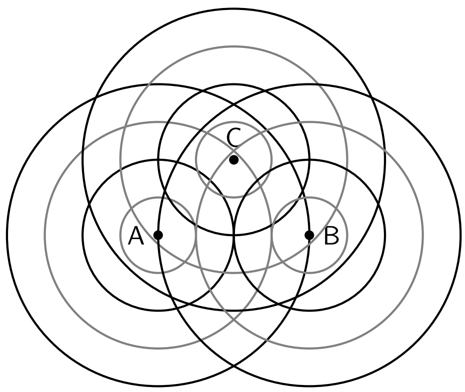
$$\begin{aligned}\sin\theta &= \frac{340}{647}\\ \theta &= \sin^{-1}\frac{340}{647}\\ \theta &= 31.7^{\circ}\end{aligned}$$

Aircraft	speed at altitude ($\mathrm{km\cdot h^{-1}}$)	speed at altitude ($\mathrm{m\cdot s^{-1}}$)	Mach Cone Angle (degrees)
Concorde	2 330	647	31.7
Gripen	2 410	669	
Mirage F1	2 573	990	
Mig 27	1 885	524	
F 15	2 660	739	
F 16	2 414	671	

Now calculate the Mach Cone Angles for the other aircraft in the table.

26.7 End of Chapter Exercises

1. In the diagram below the peaks of wavefronts are shown by black lines and the troughs by grey lines. Mark all the points where constructive interference between two waves is taking place and where destructive interference is taking place. Also note whether the interference results in a peak or a trough.



2. For an slit of width 1300 nm, calculate the first 3 minima for light of the following wavelengths:

   A blue at 475 nm

   B green at 510 nm

   C yellow at 570 nm

   D red at 650 nm

3. For light of wavelength 540 nm, determine what the width of the slit needs to be to have the first minimum at:

   A 7.76 degrees

   B 12.47 degrees

   C 21.1 degrees

4. For light of wavelength 635 nm, determine what the width of the slit needs to be to have the second minimum at:

   A 12.22 degrees

   B 18.51 degrees

   C 30.53 degrees

5. If the first minimum is at 8.21 degrees and the second minimum is at 16.6 degrees, what is the wavelength of light and the width of the slit? (**Hint:** solve simultaneously.)

6. Determine the Mach Number, with a speed of sound of 340 $\mathrm{m \cdot s^{-1}}$, for the following aircraft speeds:

   A 640 $\mathrm{m \cdot s^{-1}}$

   B 980 $\mathrm{m \cdot s^{-1}}$

   C 500 $\mathrm{m \cdot s^{-1}}$

   D 450 $\mathrm{m \cdot s^{-1}}$

   E 1300 $\mathrm{km \cdot h^{-1}}$

F 1450 $\text{km} \cdot \text{h}^{-1}$

G 1760 $\text{km} \cdot \text{h}^{-1}$

7. If an aircraft has a Mach Number of 3.3 and the speed of sound is 340 $\text{m} \cdot \text{s}^{-1}$, what is its speed?

8. Determine the Mach Cone angle, with a speed of sound of 340 $\text{m} \cdot \text{s}^{-1}$, for the following aircraft speeds:

   A 640 $\text{m} \cdot \text{s}^{-1}$

   B 980 $\text{m} \cdot \text{s}^{-1}$

   C 500 $\text{m} \cdot \text{s}^{-1}$

   D 450 $\text{m} \cdot \text{s}^{-1}$

   E 1300 $\text{km} \cdot \text{h}^{-1}$

   F 1450 $\text{km} \cdot \text{h}^{-1}$

   G 1760 $\text{km} \cdot \text{h}^{-1}$

9. Determine the aircraft speed, with a speed of sound of 340 $\text{m} \cdot \text{s}^{-1}$, for the following Mach Cone Angles:

   A 58.21 degrees

   B 49.07 degrees

   C 45.1 degrees

   D 39.46 degrees

   E 31.54 degrees

Appendix A

GNU Free Documentation License

PREAMBLE

The purpose of this License is to make a manual, textbook, or other functional and useful document "free" in the sense of freedom: to assure everyone the effective freedom to copy and redistribute it, with or without modifying it, either commercially or non-commercially. Secondarily, this License preserves for the author and publisher a way to get credit for their work, while not being considered responsible for modifications made by others.

This License is a kind of "copyleft", which means that derivative works of the document must themselves be free in the same sense. It complements the GNU General Public License, which is a copyleft license designed for free software.

We have designed this License in order to use it for manuals for free software, because free software needs free documentation: a free program should come with manuals providing the same freedoms that the software does. But this License is not limited to software manuals; it can be used for any textual work, regardless of subject matter or whether it is published as a printed book. We recommend this License principally for works whose purpose is instruction or reference.

APPLICABILITY AND DEFINITIONS

This License applies to any manual or other work, in any medium, that contains a notice placed by the copyright holder saying it can be distributed under the terms of this License. Such a notice grants a world-wide, royalty-free license, unlimited in duration, to use that work under the conditions stated herein. The "Document", below, refers to any such manual or work. Any member of the public is a licensee, and is addressed as "you". You accept the license if you copy, modify or distribute the work in a way requiring permission under copyright law.

A "Modified Version" of the Document means any work containing the Document or a portion of it, either copied verbatim, or with modifications and/or translated into another language.

A "Secondary Section" is a named appendix or a front-matter section of the Document that deals exclusively with the relationship of the publishers or authors of the Document to the Document's overall subject (or to related matters) and contains nothing that could fall directly within that overall subject. (Thus, if the Document is in part a textbook of mathematics, a

Secondary Section may not explain any mathematics.) The relationship could be a matter of historical connection with the subject or with related matters, or of legal, commercial, philosophical, ethical or political position regarding them.

The "Invariant Sections" are certain Secondary Sections whose titles are designated, as being those of Invariant Sections, in the notice that says that the Document is released under this License. If a section does not fit the above definition of Secondary then it is not allowed to be designated as Invariant. The Document may contain zero Invariant Sections. If the Document does not identify any Invariant Sections then there are none.

The "Cover Texts" are certain short passages of text that are listed, as Front-Cover Texts or Back-Cover Texts, in the notice that says that the Document is released under this License. A Front-Cover Text may be at most 5 words, and a Back-Cover Text may be at most 25 words.

A "Transparent" copy of the Document means a machine-readable copy, represented in a format whose specification is available to the general public, that is suitable for revising the document straightforwardly with generic text editors or (for images composed of pixels) generic paint programs or (for drawings) some widely available drawing editor, and that is suitable for input to text formatters or for automatic translation to a variety of formats suitable for input to text formatters. A copy made in an otherwise Transparent file format whose markup, or absence of markup, has been arranged to thwart or discourage subsequent modification by readers is not Transparent. An image format is not Transparent if used for any substantial amount of text. A copy that is not "Transparent" is called "Opaque".

Examples of suitable formats for Transparent copies include plain ASCII without markup, Texinfo input format, LaTeX input format, SGML or XML using a publicly available DTD and standard-conforming simple HTML, PostScript or PDF designed for human modification. Examples of transparent image formats include PNG, XCF and JPG. Opaque formats include proprietary formats that can be read and edited only by proprietary word processors, SGML or XML for which the DTD and/or processing tools are not generally available, and the machine-generated HTML, PostScript or PDF produced by some word processors for output purposes only.

The "Title Page" means, for a printed book, the title page itself, plus such following pages as are needed to hold, legibly, the material this License requires to appear in the title page. For works in formats which do not have any title page as such, "Title Page" means the text near the most prominent appearance of the work's title, preceding the beginning of the body of the text.

A section "Entitled XYZ" means a named subunit of the Document whose title either is precisely XYZ or contains XYZ in parentheses following text that translates XYZ in another language. (Here XYZ stands for a specific section name mentioned below, such as "Acknowledgements", "Dedications", "Endorsements", or "History".) To "Preserve the Title" of such a section when you modify the Document means that it remains a section "Entitled XYZ" according to this definition.

The Document may include Warranty Disclaimers next to the notice which states that this License applies to the Document. These Warranty Disclaimers are considered to be included by reference in this License, but only as regards disclaiming warranties: any other implication that these Warranty Disclaimers may have is void and has no effect on the meaning of this License.

VERBATIM COPYING

You may copy and distribute the Document in any medium, either commercially or non-commercially, provided that this License, the copyright notices, and the license notice saying this License applies to the Document are reproduced in all copies, and that you add no other conditions whatsoever to those of this License. You may not use technical measures to obstruct or control the reading or further copying of the copies you make or distribute. However, you may accept compensation in exchange for copies. If you distribute a large enough number of copies you must also follow the conditions in section A.

You may also lend copies, under the same conditions stated above, and you may publicly display copies.

COPYING IN QUANTITY

If you publish printed copies (or copies in media that commonly have printed covers) of the Document, numbering more than 100, and the Document's license notice requires Cover Texts, you must enclose the copies in covers that carry, clearly and legibly, all these Cover Texts: Front-Cover Texts on the front cover, and Back-Cover Texts on the back cover. Both covers must also clearly and legibly identify you as the publisher of these copies. The front cover must present the full title with all words of the title equally prominent and visible. You may add other material on the covers in addition. Copying with changes limited to the covers, as long as they preserve the title of the Document and satisfy these conditions, can be treated as verbatim copying in other respects.

If the required texts for either cover are too voluminous to fit legibly, you should put the first ones listed (as many as fit reasonably) on the actual cover, and continue the rest onto adjacent pages.

If you publish or distribute Opaque copies of the Document numbering more than 100, you must either include a machine-readable Transparent copy along with each Opaque copy, or state in or with each Opaque copy a computer-network location from which the general network-using public has access to download using public-standard network protocols a complete Transparent copy of the Document, free of added material. If you use the latter option, you must take reasonably prudent steps, when you begin distribution of Opaque copies in quantity, to ensure that this Transparent copy will remain thus accessible at the stated location until at least one year after the last time you distribute an Opaque copy (directly or through your agents or retailers) of that edition to the public.

It is requested, but not required, that you contact the authors of the Document well before redistributing any large number of copies, to give them a chance to provide you with an updated version of the Document.

MODIFICATIONS

You may copy and distribute a Modified Version of the Document under the conditions of sections A and A above, provided that you release the Modified Version under precisely this License, with the Modified Version filling the role of the Document, thus licensing distribution and modification of the Modified Version to whoever possesses a copy of it. In addition, you must do these things in the Modified Version:

1. Use in the Title Page (and on the covers, if any) a title distinct from that of the Document, and from those of previous versions (which should, if there were any, be listed in the History section of the Document). You may use the same title as a previous version if the original publisher of that version gives permission.
2. List on the Title Page, as authors, one or more persons or entities responsible for authorship of the modifications in the Modified Version, together with at least five of the principal authors of the Document (all of its principal authors, if it has fewer than five), unless they release you from this requirement.
3. State on the Title page the name of the publisher of the Modified Version, as the publisher.
4. Preserve all the copyright notices of the Document.
5. Add an appropriate copyright notice for your modifications adjacent to the other copyright notices.

6. Include, immediately after the copyright notices, a license notice giving the public permission to use the Modified Version under the terms of this License, in the form shown in the Addendum below.
7. Preserve in that license notice the full lists of Invariant Sections and required Cover Texts given in the Document's license notice.
8. Include an unaltered copy of this License.
9. Preserve the section Entitled "History", Preserve its Title, and add to it an item stating at least the title, year, new authors, and publisher of the Modified Version as given on the Title Page. If there is no section Entitled "History" in the Document, create one stating the title, year, authors, and publisher of the Document as given on its Title Page, then add an item describing the Modified Version as stated in the previous sentence.
10. Preserve the network location, if any, given in the Document for public access to a Transparent copy of the Document, and likewise the network locations given in the Document for previous versions it was based on. These may be placed in the "History" section. You may omit a network location for a work that was published at least four years before the Document itself, or if the original publisher of the version it refers to gives permission.
11. For any section Entitled "Acknowledgements" or "Dedications", Preserve the Title of the section, and preserve in the section all the substance and tone of each of the contributor acknowledgements and/or dedications given therein.
12. Preserve all the Invariant Sections of the Document, unaltered in their text and in their titles. Section numbers or the equivalent are not considered part of the section titles.
13. Delete any section Entitled "Endorsements". Such a section may not be included in the Modified Version.
14. Do not re-title any existing section to be Entitled "Endorsements" or to conflict in title with any Invariant Section.
15. Preserve any Warranty Disclaimers.

If the Modified Version includes new front-matter sections or appendices that qualify as Secondary Sections and contain no material copied from the Document, you may at your option designate some or all of these sections as invariant. To do this, add their titles to the list of Invariant Sections in the Modified Version's license notice. These titles must be distinct from any other section titles.

You may add a section Entitled "Endorsements", provided it contains nothing but endorsements of your Modified Version by various parties–for example, statements of peer review or that the text has been approved by an organisation as the authoritative definition of a standard.

You may add a passage of up to five words as a Front-Cover Text, and a passage of up to 25 words as a Back-Cover Text, to the end of the list of Cover Texts in the Modified Version. Only one passage of Front-Cover Text and one of Back-Cover Text may be added by (or through arrangements made by) any one entity. If the Document already includes a cover text for the same cover, previously added by you or by arrangement made by the same entity you are acting on behalf of, you may not add another; but you may replace the old one, on explicit permission from the previous publisher that added the old one.

The author(s) and publisher(s) of the Document do not by this License give permission to use their names for publicity for or to assert or imply endorsement of any Modified Version.

COMBINING DOCUMENTS

You may combine the Document with other documents released under this License, under the terms defined in section A above for modified versions, provided that you include in the combination all of the Invariant Sections of all of the original documents, unmodified, and list them all as Invariant Sections of your combined work in its license notice, and that you preserve all their Warranty Disclaimers.

The combined work need only contain one copy of this License, and multiple identical Invariant Sections may be replaced with a single copy. If there are multiple Invariant Sections with the same name but different contents, make the title of each such section unique by adding at the end of it, in parentheses, the name of the original author or publisher of that section if known, or else a unique number. Make the same adjustment to the section titles in the list of Invariant Sections in the license notice of the combined work.

In the combination, you must combine any sections Entitled "History" in the various original documents, forming one section Entitled "History"; likewise combine any sections Entitled "Acknowledgements", and any sections Entitled "Dedications". You must delete all sections Entitled "Endorsements".

COLLECTIONS OF DOCUMENTS

You may make a collection consisting of the Document and other documents released under this License, and replace the individual copies of this License in the various documents with a single copy that is included in the collection, provided that you follow the rules of this License for verbatim copying of each of the documents in all other respects.

You may extract a single document from such a collection, and distribute it individually under this License, provided you insert a copy of this License into the extracted document, and follow this License in all other respects regarding verbatim copying of that document.

AGGREGATION WITH INDEPENDENT WORKS

A compilation of the Document or its derivatives with other separate and independent documents or works, in or on a volume of a storage or distribution medium, is called an "aggregate" if the copyright resulting from the compilation is not used to limit the legal rights of the compilation's users beyond what the individual works permit. When the Document is included an aggregate, this License does not apply to the other works in the aggregate which are not themselves derivative works of the Document.

If the Cover Text requirement of section A is applicable to these copies of the Document, then if the Document is less than one half of the entire aggregate, the Document's Cover Texts may be placed on covers that bracket the Document within the aggregate, or the electronic equivalent of covers if the Document is in electronic form. Otherwise they must appear on printed covers that bracket the whole aggregate.

TRANSLATION

Translation is considered a kind of modification, so you may distribute translations of the Document under the terms of section A. Replacing Invariant Sections with translations requires special permission from their copyright holders, but you may include translations of some or all Invariant Sections in addition to the original versions of these Invariant Sections. You may include a translation of this License, and all the license notices in the Document, and any Warranty Disclaimers, provided that you also include the original English version of this License and the original versions of those notices and disclaimers. In case of a disagreement between the translation and the original version of this License or a notice or disclaimer, the original version will prevail.

If a section in the Document is Entitled "Acknowledgements", "Dedications", or "History", the requirement (section A) to Preserve its Title (section A) will typically require changing the actual title.

TERMINATION

You may not copy, modify, sub-license, or distribute the Document except as expressly provided for under this License. Any other attempt to copy, modify, sub-license or distribute the Document is void, and will automatically terminate your rights under this License. However, parties who have received copies, or rights, from you under this License will not have their licenses terminated so long as such parties remain in full compliance.

FUTURE REVISIONS OF THIS LICENSE

The Free Software Foundation may publish new, revised versions of the GNU Free Documentation License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns. See http://www.gnu.org/copyleft/.

Each version of the License is given a distinguishing version number. If the Document specifies that a particular numbered version of this License "or any later version" applies to it, you have the option of following the terms and conditions either of that specified version or of any later version that has been published (not as a draft) by the Free Software Foundation. If the Document does not specify a version number of this License, you may choose any version ever published (not as a draft) by the Free Software Foundation.

ADDENDUM: How to use this License for your documents

To use this License in a document you have written, include a copy of the License in the document and put the following copyright and license notices just after the title page:

> Copyright © YEAR YOUR NAME. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

If you have Invariant Sections, Front-Cover Texts and Back-Cover Texts, replace the "with...Texts." line with this:

with the Invariant Sections being LIST THEIR TITLES, with the Front-Cover Texts being LIST, and with the Back-Cover Texts being LIST.

If you have Invariant Sections without Cover Texts, or some other combination of the three, merge those two alternatives to suit the situation.

If your document contains nontrivial examples of program code, we recommend releasing these examples in parallel under your choice of free software license, such as the GNU General Public License, to permit their use in free software.